Analysis and Design Hilbert Curve Fractal Antenna Feed with Co-planar Waveguide for multi-band wireless communications

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Abstract

There are many techniques to improve the characteristic of antennas. In this work we use ideas of the fractal. The purpose of this project is to design and analyze Hilbert curve fractal antennas to get the empirical and electrical model. We use the Zealand program for simulating antennas. The antennas receive and transmit in many frequency resonances. We design a small Hilbert curve fractal antenna. We analyze this antenna by using the concept of the CPW transmission line and the mathematical definition of fractal to yield the models for Hilbert curve fractal antenna. From these models we can predict the multi resonance frequency. In the experiment we found that the least percent of difference for electromagnetics formular model with the experiment (0.4%) is lower than the least of the difference for empirical model (4.43%) because the electromagnetics model used the transmission line model while the empirical model used the numerical method. These models will be helpful for design and making Hilbert curve fractal antenna.

Keywords: Fractal antenna, Multi-band, Hilbert curve, Electromagnetics model, Coplanar waveguid feed.

1. INTRODUCTION

The term fractal was coined by the French mathematician B.B. Mandelbrot during 1970's after his pioneering research on several naturally occurring irregular and fragmented geometries not contained within the realms of conventional Euclidian geometry [1].The use of fractal geometries has significantly impacted many areas of science and engineering; one of which is antennas. Antennas using some of these geometries for various telecommunications applications are already available commercially. The use of fractal geometries has been shown to improve several antenna features to varying extents. Yet a direct corroboration between antenna characteristics and geometrical properties of underlying fractals has been missing. This research work is intended as a first step to fill this gap. In terms of antenna performance, fractal shaped geometries are believed to result in multi-band characteristics and reduction of antenna size. A quantitative link between multi-band characteristics of the antenna and a mathematically expressible feature of the fractal geometry is needed for design optimization. To explore this, we design the Hilbert curves fractal antenna that use the coplanar wave guide feed. This has been explored numerically and validated experimentally. One of the advantages of using fractal geometries in small antennas is the order associated with these geometries in contrast to an arbitrary meandering of random line segments (which may also result in small antennas). However this fact has not been used in antenna design thus far. In this work, approximate expressions for designing antennas with these geometries have been derived incorporating their fractal nature. To conclude, the research work reported here is a numerical and experimental study in identifying features of fractal shaped antennas that could impart increased flexibility in the design of newer generation wireless systems.

Several antenna configurations based on fractal geometries have been reported in recent years [2] – [4]. These are low profile antennas with moderate gain and can be made operative at multiple frequency bands and hence are multi-functional. In this work the multi-band (multifunctional) aspect of antenna designs are explored further with special emphasis on identifying fractal properties that impact antenna multi-band characteristics. Antennas with reduced size have been obtained using Hilbert curve fractal geometry. Further more, design equations for these antennas are obtained in terms of its geometrical parameters such as fractal dimension. One of the fundamental advantages of using a fractal geometry in antennas is reducing the size of a resonant antenna. This is very evident in dipole and monopole antennas using fractal Koch curves [5], and some of their modifications in the form of closed loops, and Minkowski curves [6]. The ability of these geometries to pack longer curves within relatively smaller area is the salient aspect in their use in antennas. Being a plane filling geometry, Hilbert curves can enclose longer curves for a given area than Koch curves [7].

2. Hilbert Curve Fractal antenna

2.1 Axioms L system for Hilbert Curve

The first few iterations of Hilbert curves are shown in Fig. 1. It may be noticed that each successive stage consists of four copies of the previous, connected with additional line segments. This geometry is a space-Filling curve, since with a larger iteration, one may think of it as trying to fill the area it occupies. Additionally the geometry also has the following properties: self-Avoidance (as the line segments do not intersect each other), Simplicity (since the curve can be drawn with a single stroke of a pen) and self-Similarity (which will be explored later). Because of these properties, these curves are often called an FASS curves [8].

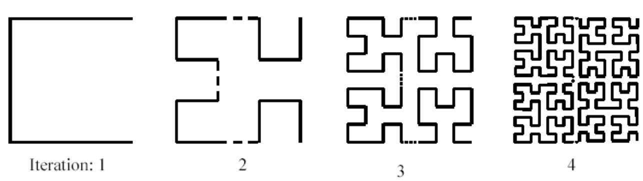


FIGURE 1: First four iterations of Hilbert curve geometry.

The segments used to connect copies of the previous iteration are shown in dashed lines. The generation algorithm of this geometry is commonly expressed in terms of L-systems. In this representation, a string of symbols with the following notations are used, leave two blank lines between successive sections as here.

(1)

(2)

where is moving forward a step, + is turn left by , - is turn right by . A recursive approach may be used to generate higher iterations (is integer = 0, 1, 2, …) of the geometry from these

(3)

(4)

2.2 Antenna Configurations Using Hilbert Curves

It would be interesting to study the properties of a new antenna with reference to various existing, and more familiar antennas. In this context, a schematic of the thought process leading to the Hilbert curve antenna. The half-wave meander line antenna is resonant when the arms are approximately quarter wavelength long. The biconical antenna is a broadband variant for the common dipole [9]. This antenna can even be simulated with wires along its periphery. Puente et al [2] have used a bowtie as the base model for explaining the properties of the Sierpinski gasket fractal antenna with multi-band radiation characteristics.

A conventional coplanar waveguide (CPW) on a dielectric substrate consists of a center strip conductor with semi-infinite ground planes on either side as shown in Fig 2. This type of antenna offers several advantages over microstrip line. It simplifies fabrication, facilitates easy shunt as well as series surface mounting of active and passive devices, eliminates the need for via holes and reduces radiation loss. In addition a ground plane exists between two adjacent lines; hence cross talk effects between them are very week. As a result, CPW circuits can be made denser than microstrip circuits [9].

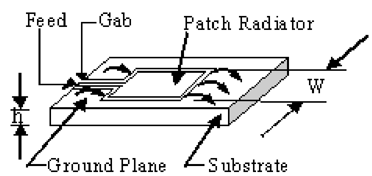
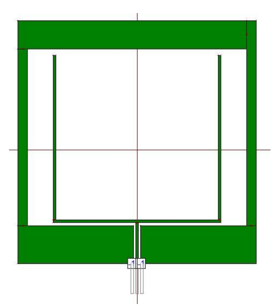


FIGURE 2: CPW antenna

CPW antennas are designed by using the IE3D program. This program has high efficiency, accuracy and low cost simulation tools. The proposed antenna is excited by a CPW line of 50 Ω and is fabricated on a FR4 substrate with a thickness (h) of 1.6 mm and relative permittivity ( ) of 4.4. The two ground planes are placed symmetrically on each side of the CPW line. Design the fundamental resonance frequency at 1.8 GHz for the first stage of Hilbert curve antenna shown in Fig 3.



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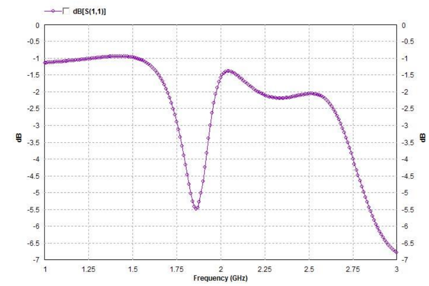
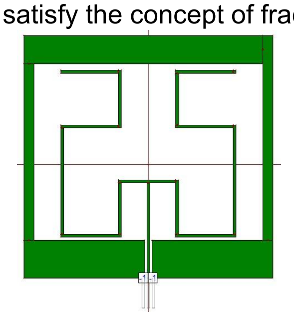


b)

FIGURE 3 : a) First stage of Hilbert curve geometry. b) the return loss of this stage

We design the next stage of this Hilbert curve antenna(stage 2, 3,and 4) and study the return loss response of each stage that shown in the Fig. 4,5 and 6 respectively. We found that the number of resonance frequency increase that satisfy the concept of fractal antenna.



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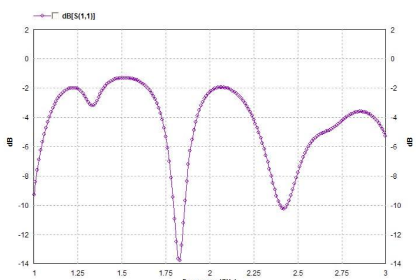
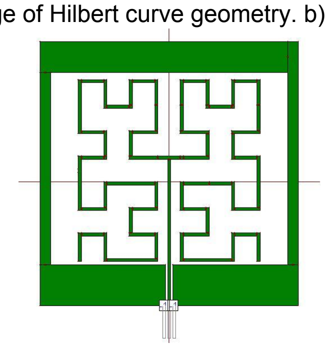


b)

FIGURE: 4 a) second stage of Hilbert curve geometry. b) the return loss of this stage



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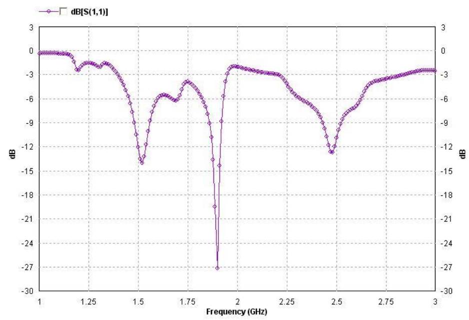
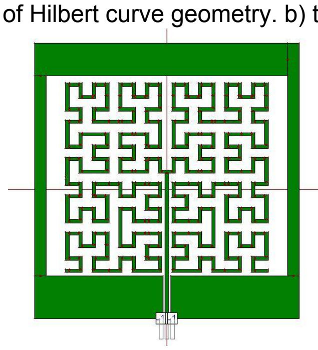


b)

FIGURE: 5 a) third stage of Hilbert curve geometry. b) the return loss of this stage



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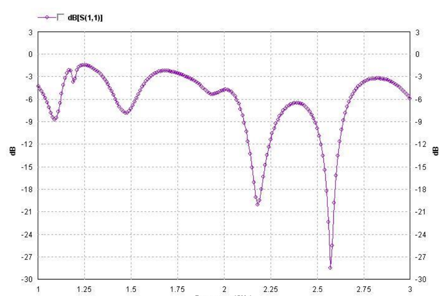

b)

FIGURE: 6 a) fourth stage of Hilbert curve geometry. b) the return loss of this stage

3. Result

We made this Hilbert Curve antenna by using the fabrication on the FR-4 substrate that shown in Fig. 7. The proposed antennas have been tested using a calibrated vector network analyzer. Measured result of the return loss(S_{11}) compared with the simulation is shown in Fig. 8.

FIGURE: 7 The photograph of proposed Hilbert curve antenna at stage 4

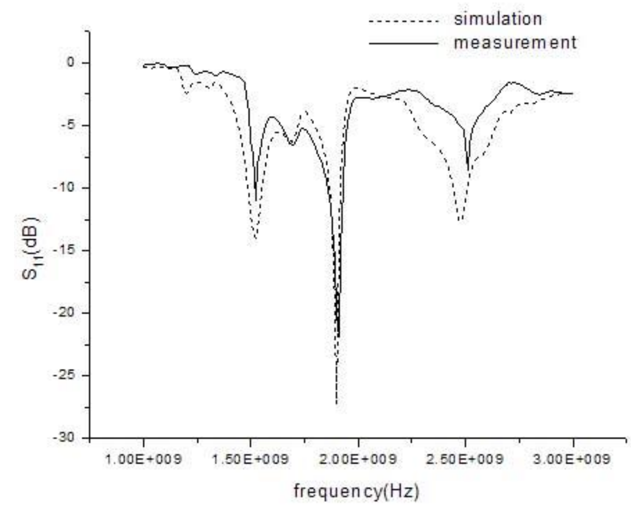

FIGURE: 8 Measured result of S11 compared with the simulation

The far-field radiation patterns of the proposed antenna have been measured by connecting transmitting antenna to the frequency sweep generator and connecting the receiving antenna to the spectrum analyzer. Measured results of the patterns had been plotted in Fig 9.

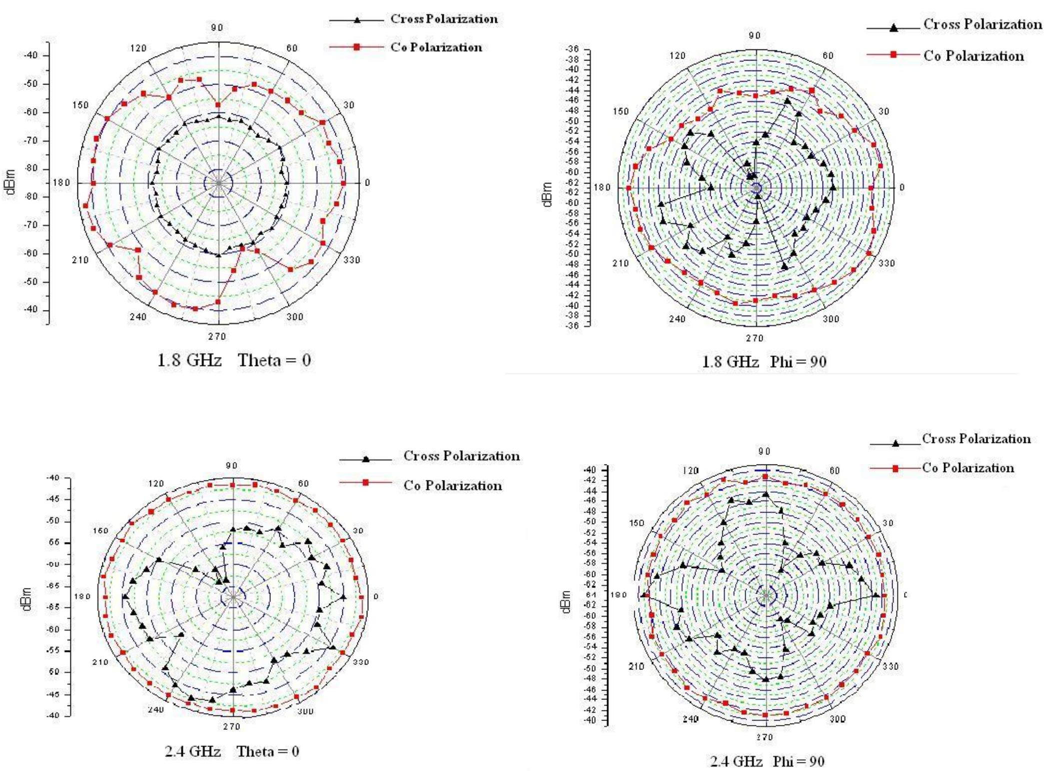

FIGURE: 9 Measured result of the patterns for the proposed CPW antennas

4 Analyze and Design formula

4.1 EMPIRICAL MODEL

It is now possible to obtain approximate design equations for this type of antenna. The approach for the design formulation is based on that followed for resonant meander line antennas [10].

The resonant frequencies are obtained through the above formulation by use the numerical method. We can find the empirical model equation by using the basic concept of antenna that state the resonance frequency vary inversely with the size of the antenna as shown in the Fig. 10

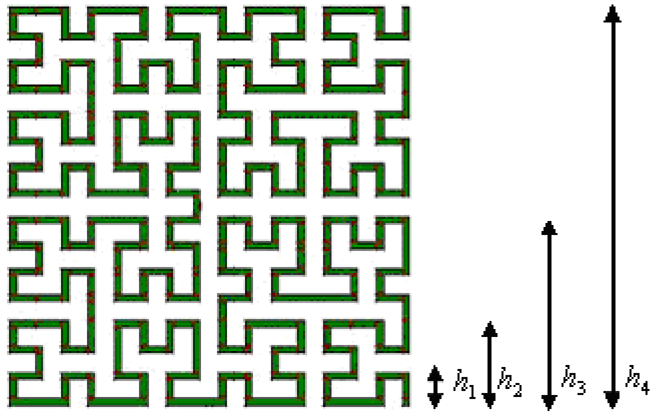

FIGURE: 10 Dimension of Hilbert Curve Fractal Antenna iteration 4 at the first fourth resonance frequency.

From this design HCFA, the dimensions of antenna are h_4=88 mm, h_3=44 mm, h_2=22 mm, and h_1=7.266 mm. This antenna wills resonance at 0.84 GHz, 1.52 GHz 1.9GHz and 2.48 GHz that satisfy the size of antenna as shown in the table 1

n(band)	f_n(GHz)	f_{n+1}/f_n (δ)	h_n/λ_n
1	0.840	1.803	0.2464
2	1.520	1.254	0.2229
3	1.900	1.302	0.14
4	2.480	1.381	-

TABLE:1 The dimension of Hilbertcurve antenna at difference Resonance frequency

From the data we found that the ratio of the next stage resonance frequency to this stage will converge to some number (1.38) that we call log period and the ratio of the height to the resonance wavelength seem to be constant about 0.21(average of 0.2464, 0.2229, 0.14) that we use this coefficient in the proposed model. Finally the data has been analyzed resulting in an empirical model formula at resonance stage n

(5)

where f_n = the resonance frequency at stage n

c= speed of electromagnetic wave= m/s
h = the highest dimension size of antenna
n = integer number at stage n (1,2,3,.,.)
δ = log period = 1.38

The resonance frequency from above model and simulation are compared with simulation. A table of comparison is given below (Table 2). From the table, the percent of difference will be smaller at the higher resonance frequency. These antennas are fabricated on a FR4. These results show a reasonable match between the simulation and empirical model, and hence it is concluded that the above formulation may be used as an empirical design equation for antennas of this type.

4.2 Electromagnetics Model

It is now possible to obtain approximate design equations for this type of antenna. The approach for the design formulation is based on that followed for resonant meander line antennas [10]. In this approach, the inductances of the turns of the meander line are calculated, considering them as short circuited parallel-two-wire lines. The self inductance of an imaginary straight line connecting all these turns is then added to this to get the total inductance. This is then compared with the inductance of a regular half wavelength meander line. Since meander line antennas with approximately half wavelength are resonant (their capacitive and inductive reactances cancel each other)[11], and assuming that the input capacitive reactance for a meander line antenna remains unchanged by reducing its apparent length by introducing turns, the resonant condition for this antenna is derived. The approach reported in [12] for the meander line meander line antenna can readily be extended for the Hilbert curve antenna (HCA). The definition of self inductance of a straight line for meander line antenna is replaced here with the total inductance of the line segments otherwise unaccounted (not forming short-circuited parallel wire sections). Another important assumption is that the capacitances of the meander line configurations remain the same in all cases. For an HCA (Fig. 10) with outer dimension of and order of fractal iteration n, the length of each line segment d is given by

(6)

The number of short circuit terminations for parallel wire section be founded that

(7)

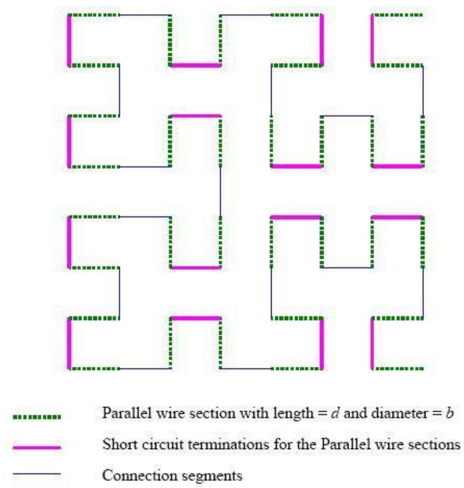

FIGURE: 11 Composition of a HCA with iteration order 3. The short-circuited parallel wire sections and connection wire sections are shown separately.

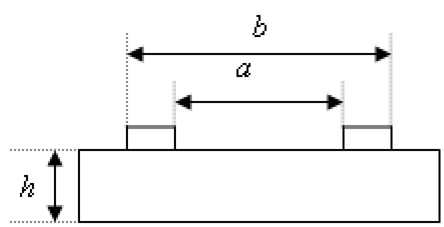

FIGURE:12 Cross section of the Coplanar Waveguide Hilbert Curve Fractal Antenna.

The segments not forming the parallel wire sections amount to a total length is

(8)

The approach we introduce to derive the condition for the resonant properties of Hilbert curve antennas printed on a dielectric substrate, is to consider sections of the strip as terminated parallel strip transmission lines. The characteristic impedance of two parallel strips of negligible thickness (t) printed on a dielectric of height (h), and dielectric constant , as shown in Fig.11 in terms of complete elliptic integral of the first kind (K) is given by [12]:

(9)

where the effective dielectric constant is

and the pure inductance is

(10)

The self inductance due to a straight line of length s as

(11)

Substituting (9) in (10) and using (11), the total inductance is therefore

(12)

It should however be noted that regular meander line antennas resonate when the arm length is a multiple of quarter wavelength. Thus, by changing the resonant length related terms on the RHS of equation, we can obtain all the resonant frequencies of the multi-band HCA. Therefore the first few resonant frequencies of the HCA can be obtained from the formula model:

(13)

where k is an odd integer (1,3,5,…) and

=propagation characteristics of the transmission line.

The resonant frequencies were obtained through the above formulation by use the numerical method to iterate the frequencies that make the LHS equal RHS. The resonance frequency from above model and simulation were compared with experiments. A table of comparison is given below (Table 2). From the table 2, these models will be more accurate at the higher frequency. These antennas are fabricated on a FR4. These results show a reasonable match between the two, and hence it is concluded that the above formulation may be used as an empirical design equation for antennas of this type.

	Resonant Frequency(GHz)

	f_{r1}	f_{r2}	f_{r3}
Simulation	1.52	1.90	2.48
Empirical Model (Percent of Difference)	1.36 (10.52%)	1.87 (1.57%)	2.59 (4.43%)
Electromagnetics Model (Percent of Difference)	1.45 (4.6%)	1.96 (3.15%)	2.47 (0.4%)

TABLE: 2 Comparison of formulation with experimental simulation results for Hilbert curve antennas printed

5. Conclusion

In this work the development of antennas using Hilbert curve fractal geometry is presented. Some of the numerical results are validated through experiments and we found that the least percent of difference for this empirical model is 4.43%. The advantage of numerical method for this work is easy because when we consider empirical model, this model gives the higher percentage of difference than the electromagnetics model that gives the least percentage of difference for this model(0.4 %). These models will be helpful for design Hilbert curve fractal antennas. The numerical results presented in this research indicate that further reduction in resonant frequency is possible for Hilbert curves. A patch configuration is also explored. However, the antenna characteristics in this configuration are found to be dictated by the outer dimensions.

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